Proof Nets of MALL

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Proof Nets Requirements

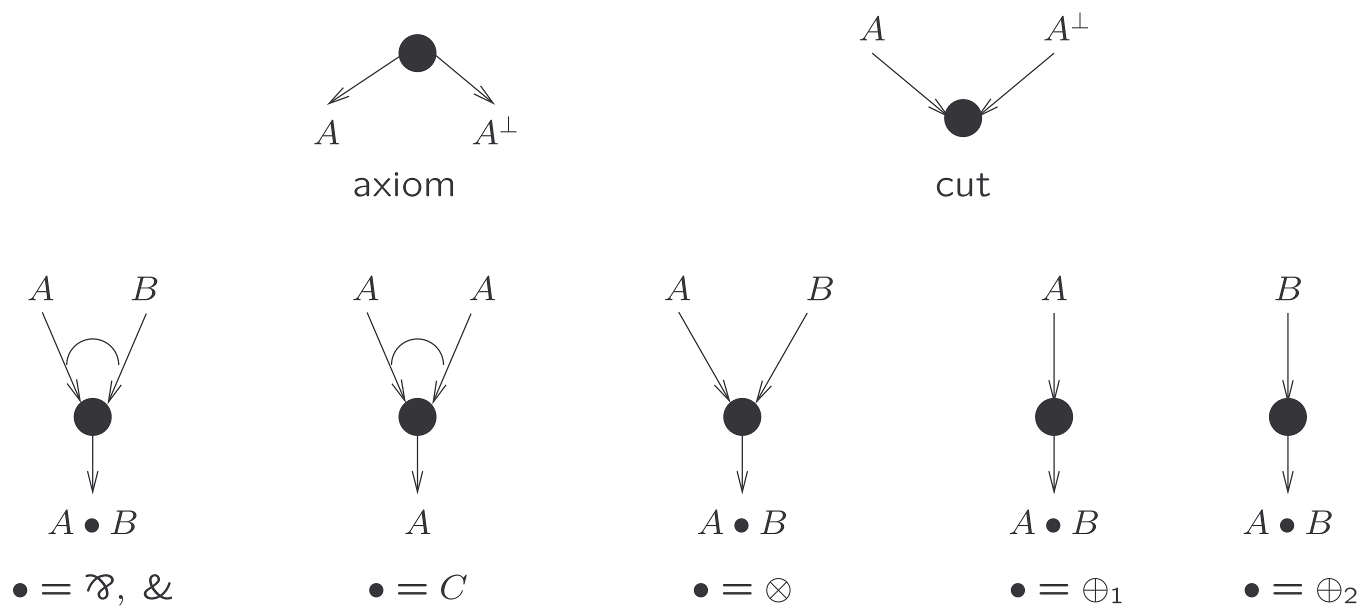
- **proof structures** must be simple (like for MLL), without special conditions (like “weights”, “resolution condition”,...);

- **cut reduction** must be local (i.e., local reduction steps), terminating and confluent;

- **correctness criterion** must be stable under cut reduction, adequate and sequentializable.

Proof Structures (PS)

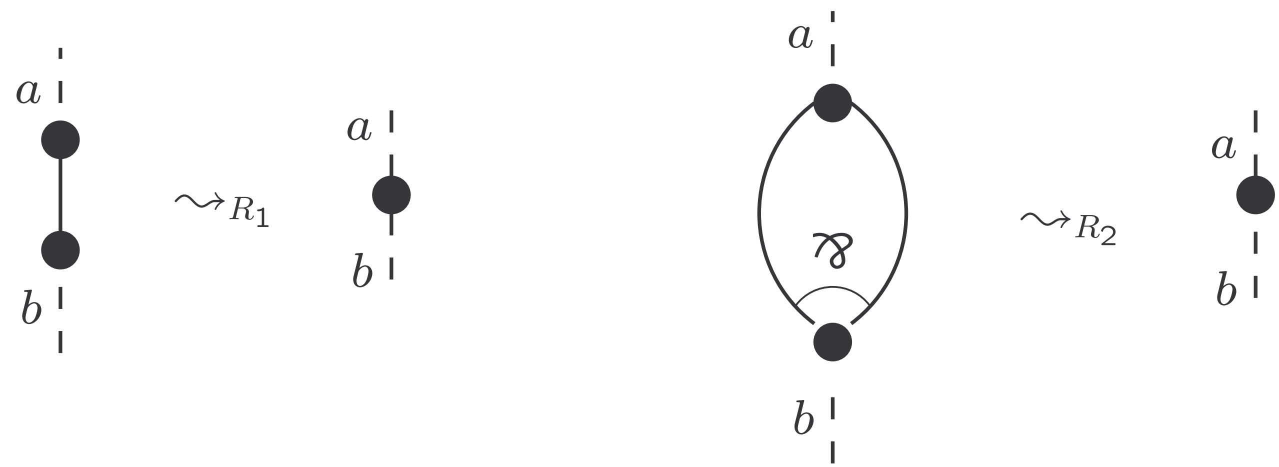
A PS is an oriented graph built on *links* s.t. each link is the conclusion of exactly one link and the premise of at most one link; pending edges are called the *conclusions* of PS.



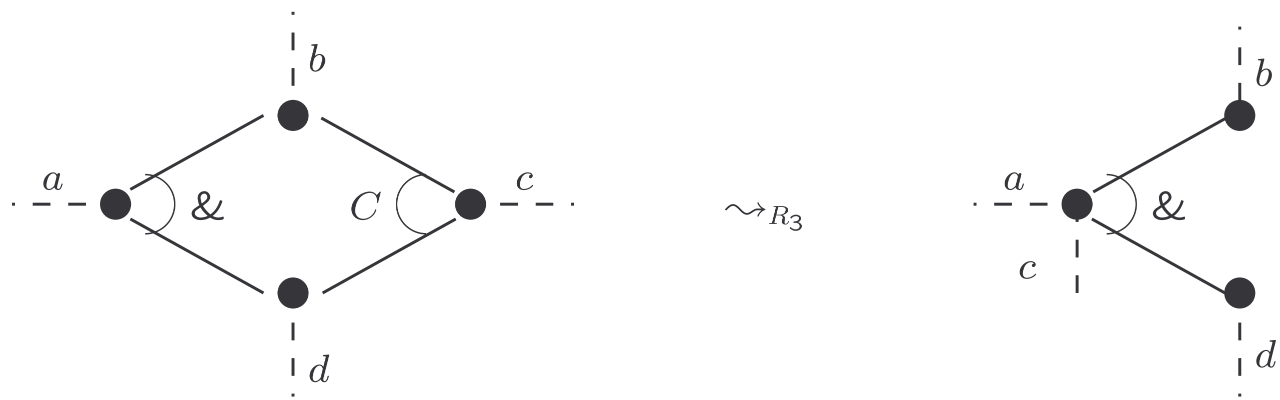
Proof Nets (PN)

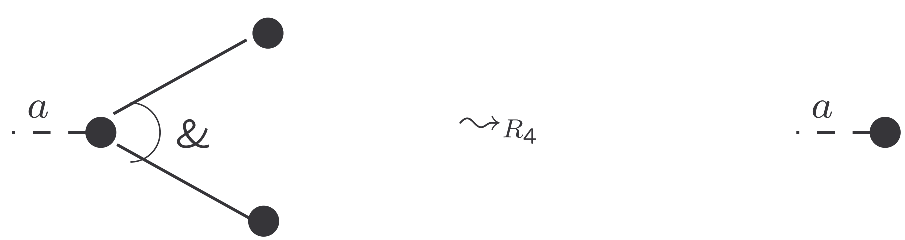
A PS with an unique conclusion is *correct*, i.e,, it is a *proof net*, if it retracts to a point ($\bullet$) by iteration of the following *retraction rules*.

Multiplicative Retraction Rules

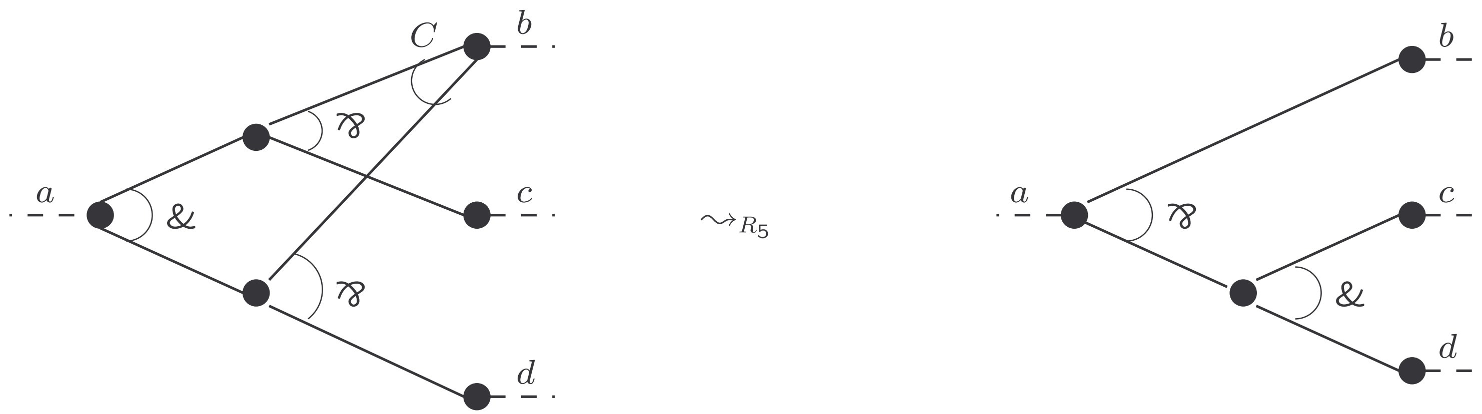


Additive Retraction Rules





Distributive Retraction Rule



reflects the $\Rightarrow$-part of the distributive law:

$$(b⅋c)\&(b⅋d) \Leftrightarrow b⅋(c\&d)$$

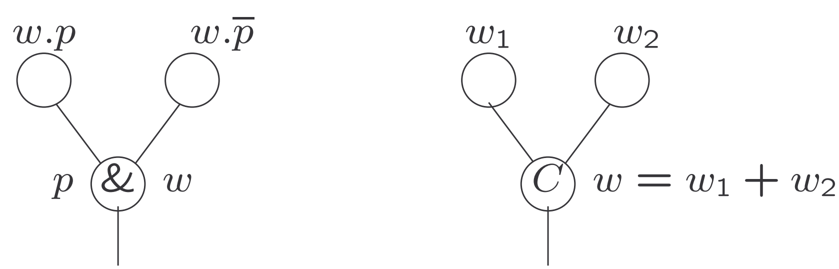
Adequacy and Sequentialization

- Adequacy
  We may map a sequent proof of Γ in to a proof net of Γ.
  (**proof**: by induction on the length of the sequent proof)

- Sequentialization
  We may map a proof net of Γ in to a sequent proof of Γ.
  (**proof**: consequence of the fact that each retractable proof structure is also a proof net *à la Girard*. It is a *non injective mapping*)

Weighted Cut Reduction

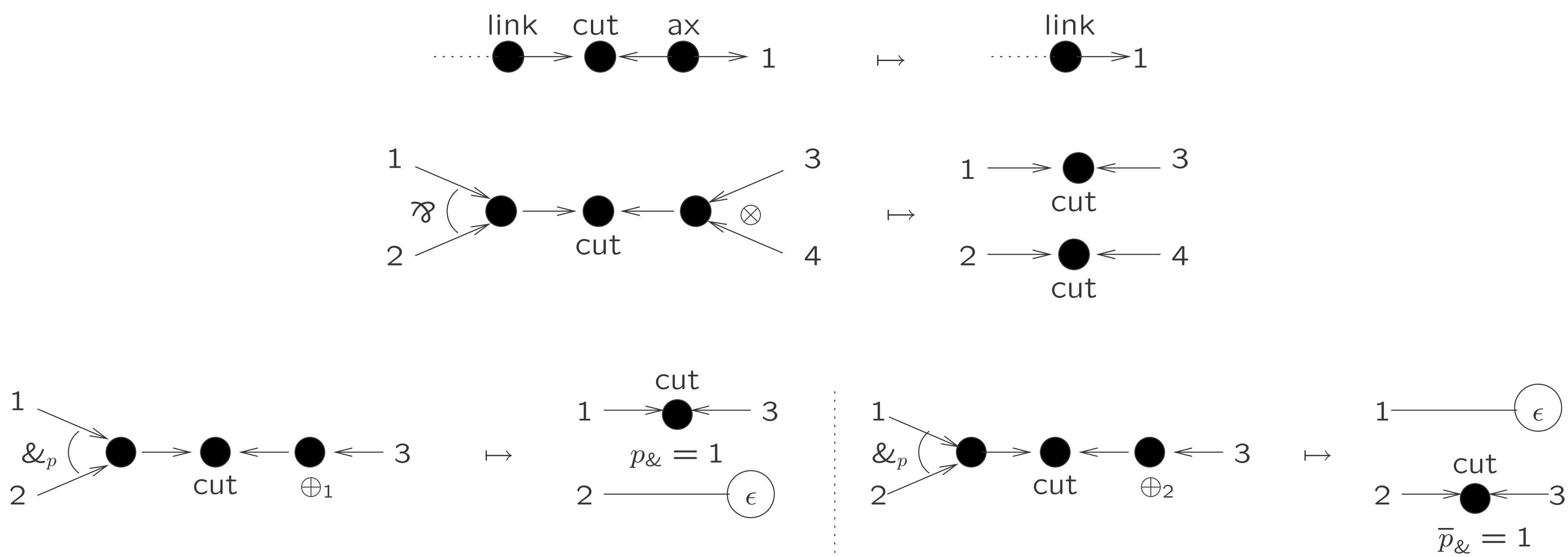
PS are weighted as follows:
- we associate different *eigen weights*, $p_{\&}, q_{\&}, ...$, to each &-link;
- we associate a *weight*, a product of (negation of) boolean variables $(p, \overline{p}, q, \overline{q}...)$ to each link, with the constraint that two links have the same weight if they have a common edge, except when the edge is the premise of a & or C link:



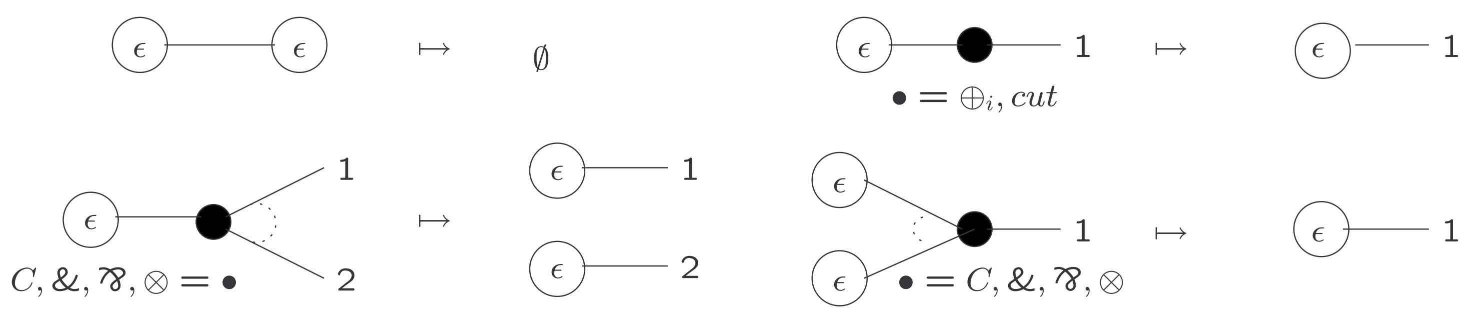
if p does not occur in w with $w_1.w_2 = 0$

- *conclusion nodes* have weight 1

Logical Reduction

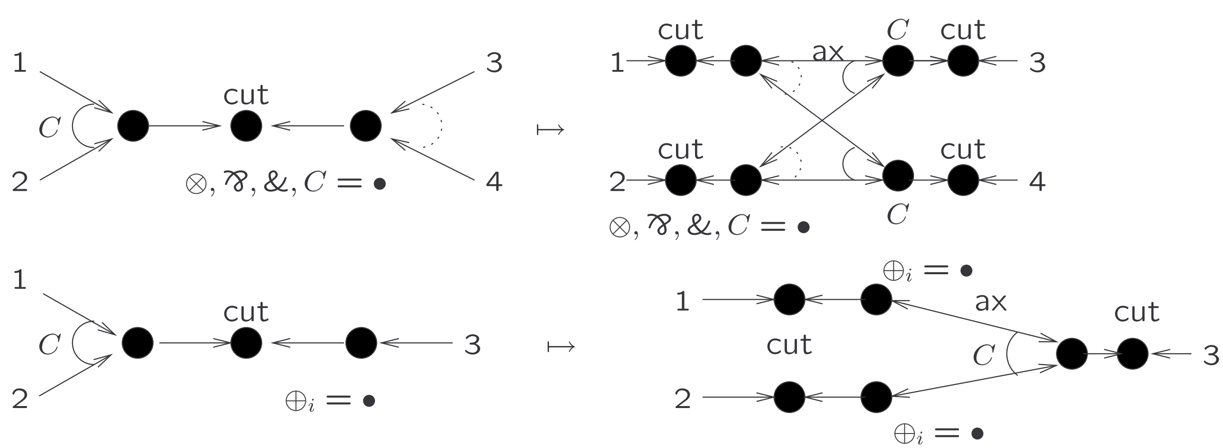


Structural Reduction: slice erasing

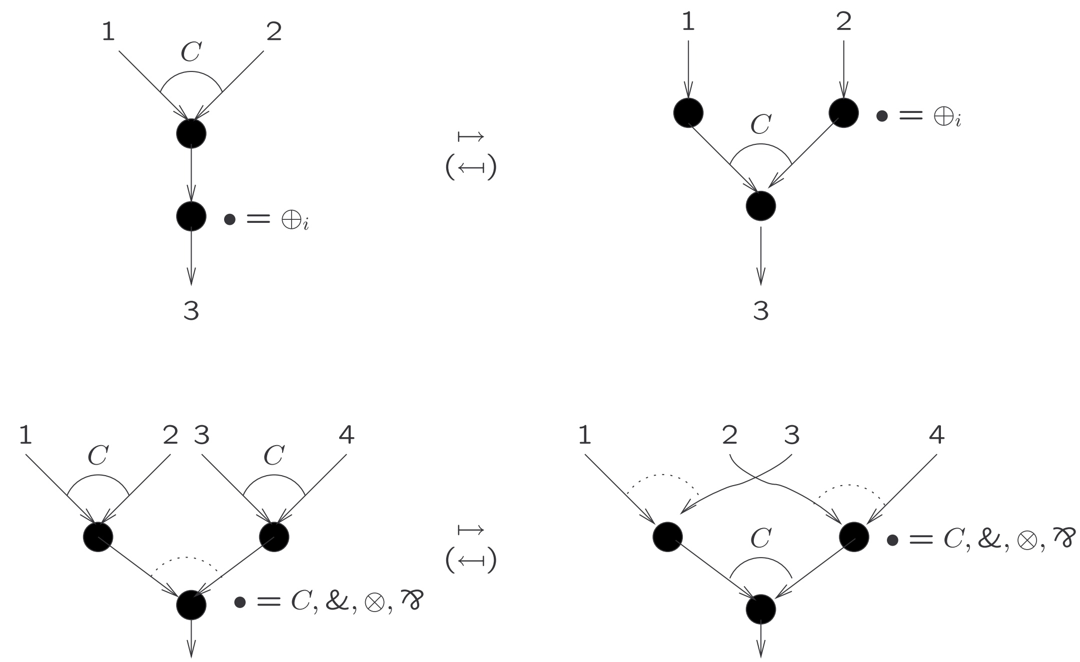


corresponding to erasing the slice with $p_{\&} = 0$ or $\overline{p}_{\&} = 0$

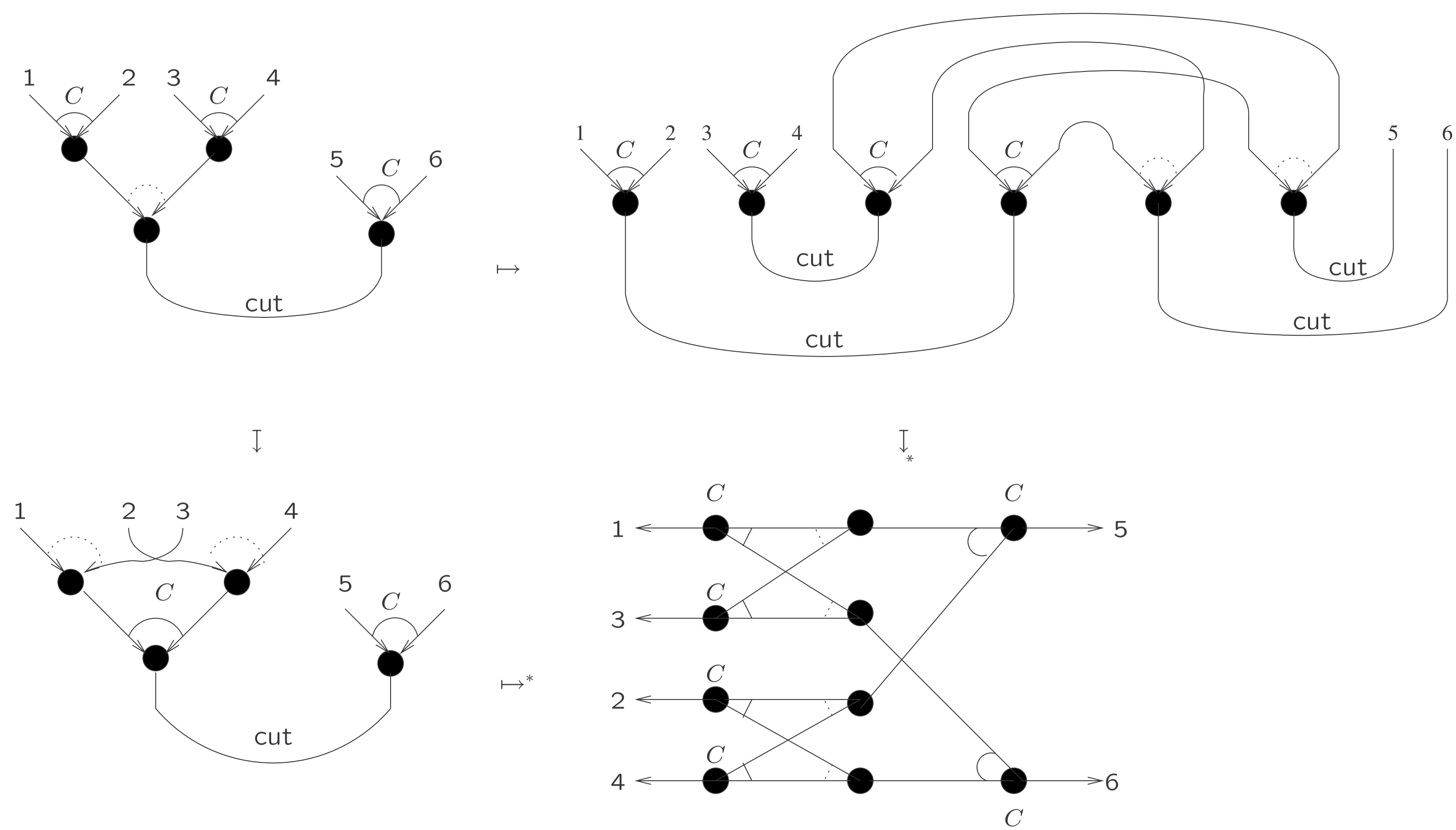
Commutative Reduction



Structural Reduction: congruence



Confluence of Reduction: crucial case

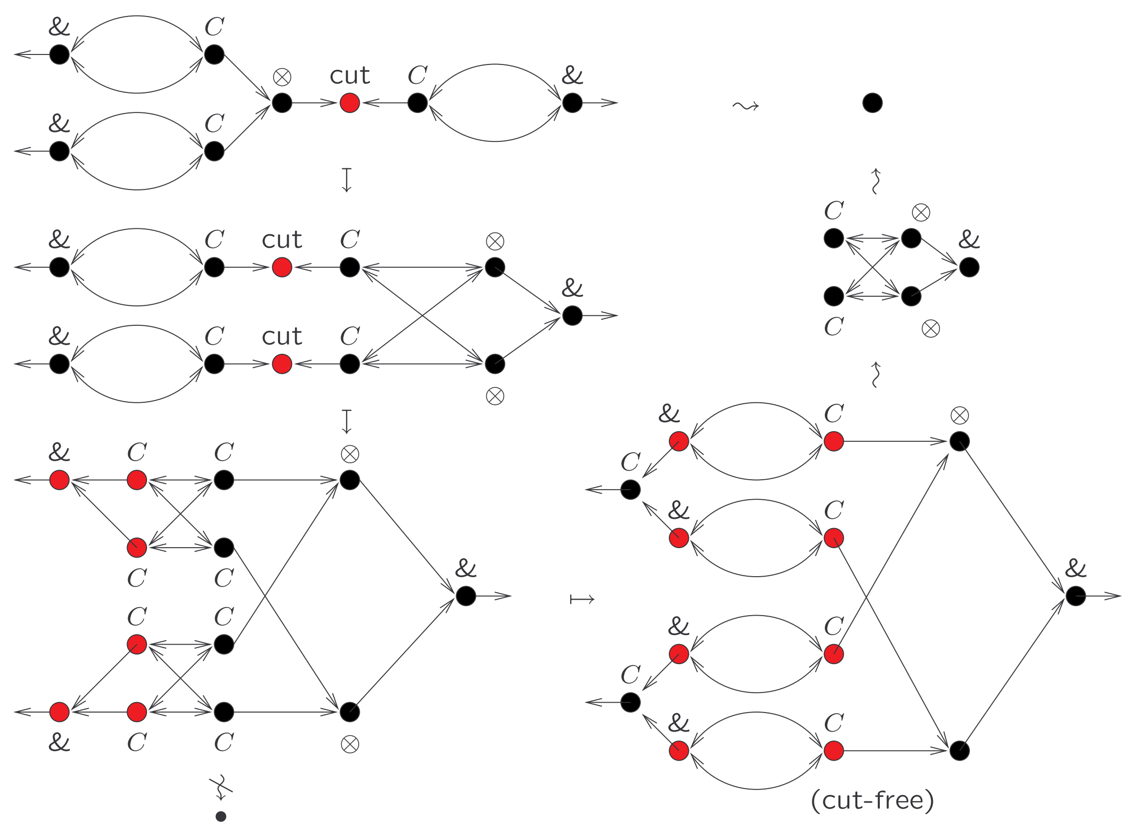


Asymptotic Stability of Correctness via Reduction

$$(\pi \leadsto^{*} \bullet) \;\wedge\; (\pi \mapsto \pi') \;\Rightarrow\; \exists \pi''.(\pi' \mapsto^{*} \pi'') \;\wedge\; (\pi'' \leadsto^{*} \bullet)$$

$$\begin{array}{ccccc}
(\bullet & {}^{*}\!\!\leftsquigarrow & \pi) & \mapsto & \pi' \\
 & & & \searrow^{*} & \downarrow^{*} \\
 & & & & (\pi'' \leadsto^{*} \bullet)
\end{array}$$

Example



(asymptotic correctness stability)

Future Work

- try to getting rid of weights in the cut reductions

- a semantical investigation of some crucial reduction rules